NATIONAL OPEN UNIVERSITY OF NIGERIA

SCHOOL OF SCIENCE AND TECHNOLOGY

COURSE CODE: PHY 456

COURSE TITLE: NUCLEAR REACTOR PHYSICS

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PHY 456: NUCLEAR REACTOR PHYSICS

COURSE GUIDE

NATIONAL OPEN UNIVERSITY OF NIGERIA

Contents

INTRODUCTION

Nuclear Reactor Physics is a very interesting course to study, The prerequisite to this course acquainted you with the basic concepts which we shall be treating here and have served as a veritable stepping stone to the more advanced concepts herein.

While the content and scope of the prerequisites are expected to have developed in you an enquiring attitude towards this course, you shall see that day to day applications abound and Nuclear Reactors represent an exciting field with the prospects of abundant and almost unlimited energy in this present world of dwindling fossil fuel reserves which hitherto serve as a driving source for today's energy needs.

We shall of course commence with the underlying concepts, principles, rules and laws behind this technology; some of which you no doubt have become familiar with as you are well aware of the awesome – albeit destructive energy output of the Atomic bomb. Let the author point out however that emphasis is laid on more constructive peacetime applications of Nuclear energy.

When armed with new insight driveable upon successful completion of this course, you will be further strengthened in your understanding of the underlying principles behind the practical applications of Nuclear energy through Nuclear reactors and you will be able to proffer your own solutions to technical questions frequently encountered in research, development and the application of this technology and gradually acquire the confidence required to discuss professionally all of the concepts treated in PHY 456.

THE COURSE

PHY 456 Nuclear Reactor Physics

This course, PHY 456 – Nuclear Reactor Physics comprises a total of five Units arranged into Three modules as listed below with a final section on Solutions and Answers to questions presented in the Units studied:

Module 1 is composed of 2 Units

Module 2 is composed of 2 Units

Module 3 is composed of 1 Unit

Module 1 comprised of two units shall devote Unit 1 to Neutron Interactions and Cross Sections which are a subset of Neutron Physics while in Unit 2 we shall discuss Thermalization with special focus on Neutron Moderation and the Passage of a Beam of Neutrons through a Moderating Material both of critical interest in the implementation of Nuclear Reactors.

Module 2 shall visit the subject of Transport and Diffusion of Neutrons and here you shall understand the pertinent equations to Neutron mobility through matter and learn both Fick's Law and the Equation of Continuity in Unit 3. Unit 4 will explain the types of Nuclear reactions; Fission reactions and Fusion reactions, and here also we shall be discussing what is meant by the criticality of a reactor.
The final module which comprises only one Unit will lay bare to you the practical applications of what you must have earlier learnt by describing to you the different types of Nuclear reactors, their designs, their merits and demerits; what Thermal reactors are, and the conditions which necessitate the design and construction of Breeder reactors in order to enrich Nuclear Fuel. The content of Unit 5 lists out and describes the Components of Nuclear reactors and how they relate to each other.

Finally, answers are provided for the many questions asked in the earlier portions of this course.

COURSE AIMS AND OBJECTIVES

The aim of PHY 456 is to provide you a platform for the understanding of the principles involved in the practical application of Nuclear energy through the development and operation of Nuclear Reactors. Because Nuclear energy is highly destructive, this course also strives to show you in quantitative terms how this energy can be controlled.

After you have worked tirelessly on this course, you should undoubtedly have been provided a unique platform upon which you should be able to:

- Explain neutron dynamics
- Explain interaction of neutrons with nuclei
- Explain the slowing down of neutrons in materials
- Explain the choice of various materials for slowing down processes
- Explain the behaviour of neutrons in a reactor
- Explain the physics of neutrons in a reactor in terms of the equation of continuity and diffusion equation
- Deduce the diffusion length of neutrons and the buckling factor of a reactor
- Explain how nuclear fission reaction produces neutrons from its chain reaction.
- Explain nuclear fusion as a thermonuclear reaction
- Explain the criticality of a reactor

- Explain the different kinds of nuclear reactors
- Identify the components of a nuclear reactor
- Explain the function of each of the components of a nuclear reactor

WORKING THROUGH THE COURSE

PHY 456 is easy to understand because the principles and theories are presented in simple language with lots of supportive illustrations. This simplicity should not be taken for granted however, and it is strong recommended that you should never assume that you already know the concepts therein, but study them carefully.

The author suggests that you spend quality time to read, as well as to relate what you have read to current events around the world where Nuclear reactors are in operation; particularly the long term environmental hazards that accompany Nuclear Reactor disaster.

You should take full advantage of the tutorial sessions as they represent an invaluable opportunity for you to “rub minds” with your peers – and this represents a valuable feedback channel as you have the opportunity of comparing and personally scoring your progress with your course mates.

COURSE MATERIAL

Prior to the commencement of this course you will be provided course material which will comprise your Course Guide as well as your Study Units. You will also be provided a list of recommended textbooks which you are expected to acquire and which shall be an invaluable asset to complement your course material. While they are strongly recommended, these textbooks are not mandatory.

STUDY UNITS

Now let us take a look at the study units which are contained in this course below. First you will observe that there are three modules which comprise two units each, except for module 3 which has only one Unit. Secondly you will see that the organisation of the content represents a logical flow from module 1 which treats the fundamental concepts of Neutron physics through to the concluding Unit where practical applications are discussed. Would you like us to take a closer look at what we shall be learning below?

- Neutron physics
- Neutron interactions

- Cross Sections
- Thermalization
- Neutron Moderation
- Passage of a Beam of Neutrons through a Moderating Material
- Transport and Diffusion Equation
- Fick's Law
- Equation of Continuity
- Nuclear Reactions
- Nuclear Fission
- Thermonuclear reaction (nuclear fusion)
- Criticality of a Reactor
- Nuclear Reactor
- Classification of Nuclear Reactors
- Thermal Reactors
- Breeder Reactors
- Components of nuclear reactors

TEXTBOOKS

It is recommended that you acquire the most recent editions of the recommended textbooks for your further reading.

- W. Greiner and J.A. Maruhn, Nuclear Models by Springer.
- R. Gautreau and W. Savin, Schaum's outline of theory and problems of Modern Physics, 1999 edition.

ASSESSMENT

It is standard NOUN practice to assess your performance partly through Tutor Marked Assessment which you can refer to as TMA, and partly through the End of Course Examinations.

TUTOR MARKED ASSIGNMENT

TMA is basically Continuous Assessment and accounts for 30% of your total score. During the study of this course, you will be given 4 Tutor Marked Assignments and of which you must compulsorily answer three of them to qualify to sit for the end of year examinations. The Tutor Marked Assignments will be provided by your Course

Facilitator and upon completing the assignments; you must return them back to your Course Facilitator within the stipulated period of time.

END OF COURSE EXAMINATION

You must sit for the End of Course Examination as these accounts for 70% of your score upon completion of this course. You will be notified in advance of the date, time and the venue for the examinations which may, or may not coincide with National Open University of Nigeria semester examination.

SUMMARY

Each of the three modules of this course has been carefully tailored to stimulate your interest in a specific area of Nuclear Reactor Physics in. In particular; the progression from Module 1 which treats Neutron physics through Module 2 and concluding with module 3 that explains practical Nuclear Reactor designs will will enable you to understand the content of the course with relative ease and will also facilitate the translation of abstract theoretical concepts to real world subsystems and systems which you can relate to.

This coursework provides you invaluable insight into the discovery, development and the functioning of Nuclear Reactors and the simple concepts which constitute the building blocks upon which the complex constructs of a functional Nuclear Power station is built and which offers abundant clean energy (Nuclear Fusion) capable of transforming the world which we live in today.

You will upon completion of this course be able to discuss the knowledge space contained within with confidence, and will also be able to proffer realistic solutions and answers to everyday questions that arise.

Ensure that you have enough referential and study material available and at your disposal as this course will change your perception of the world around you in more ways than one.

On this note;

I wish you the very best as you seek knowledge - always bearing in mind that Albert Einstein also trod this path once..

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UNIT 1 NEUTRON PHYSICS

CONTENTS

1.0 INTRODUCTION

The design of all nuclear systems- reactors, radiation shield, isotopic generators, and so on- depends fundamentally on the way in which nuclear radiation interacts with matter. In this unit these interactions are discussed for neutrons only with energies up to 20MeV. Most of the radiation encountered in practical nuclear devices lies in this energy region.

1.1 OBJECTIVES

After going through this unit, you will be able to:

- Explain neutron dynamics
- Explain interaction of nuclear radiations (neutrons) with nuclei

1.2 MAIN CONTENT

1.2.1 NEUTRON INTERACTIONS

It is important to recognize at the outset that since neutrons are electrically neutral, they are not affected by the electrons in an atom or by positive charge of the nucleus. As a consequence, neutrons pass through the atomic electron cloud and interact directly with the nucleus. In short, neutrons collide with nuclei, not with atoms.

Neutrons may interact with nuclei in one or more of the following ways:

(i) *Elastic scattering:* in this process, the neutrons strike the nucleus, which is almost always in its ground state, the neutron reappears, and the nucleus is left in the ground state. The neutron in this is said to have been *elastically* scattered by the nucleus. In this notation of nuclear reactions, this interaction is abbreviated by the symbol (n,n).

(ii) *Inelastic scattering:* this process is identical to elastic scattering except that the nucleus is left in an excited state. Because energy is retained by the nucleus, this is clearly an endothermic interaction. Inelastic scattering is denoted by the symbol (n,n’). The excited nucleus decays by the emission of γ-rays. In this case, since these γ-rays originate in inelastic scattering, they are called inelastic γ-rays.

(iii) *Radioactive capture:* here the neutron is captured by the nucleus, and one or more γ-rays – called capture γ-rays- are emitted. This is an exothermic interaction and is denoted by (n, γ). Since the original neutron is absorbed, this process is an example of a class of interactions known as *absorption reactions.*

(iv) *Charged-particle reactions:* Neutrons may also disappear as the result of absorption reactions of the type (n,α) and (n,p). Such reactions may be either exothermic or endothermic.

(v) *Neutron-producing reactions:* Reactions of three type (n,2n) and (n,3n) occur with energetic neutrons. These reactions are clearly endothermic since in the (n,2n) reaction one neutron, and in the (n,3n) reaction two neutrons are extracted from the struck nucleus. The

(n,2n) reaction is especially important in reactors containing heavy water or beryllium since

2_H *and* 9_{Be} have loosely bound neutrons which can easily be ejected.

(vi) *Fission:* Neutrons colliding with certain nuclei may cause the nucleus to split apart, i.e., undergo fission. This reaction is the principal source of nuclear energy for practical applications.

SELF ASSESSMENT TEST 1

(i) List and briefly explain the different ways by which neutrons can interact with nuclei.

1.2.2 CROSS SECTIONS

The extent to which neutrons interact with nuclei is described in terms of quantities known as *cross sections.* These are defined by the following type of experiment. Suppose that a beam of monoenergetic (single energy) neutrons impinge upon a thin target of thickness ‘X’ and area ‘a’ as shown in Fig. 1. If there are n neutrons per cm^3 in the beam and v is the speed of the neutrons, then quantity

$$I = nv \qquad (1.1)$$

is called the *intensity* of the beam. Since the neutrons travel the distance v cm in 1 sec, all of the neutrons in the volume va in front of the target will hit the target in 1 sec. Thus, $nva = Ia$ neutrons strike the entire target per second, and it follows that $Ia\ a = I$ is equal to the number of neutrons striking the target per cm^2/sec.

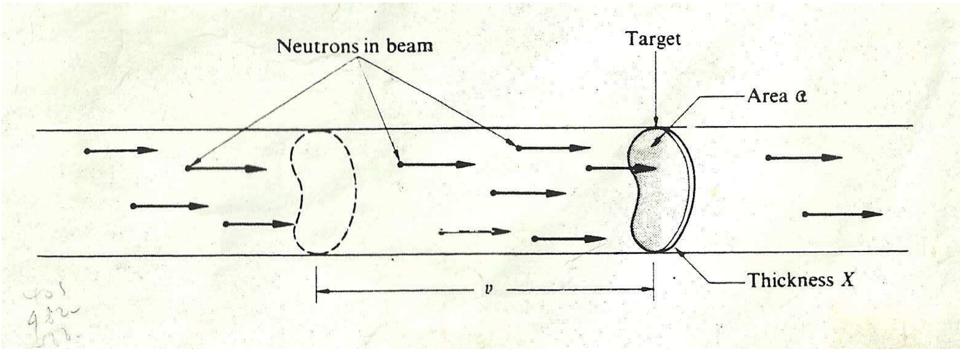


Fig.1 Neutron beam striking a target

Since nuclei are small and the target is assumed to be thin, most of the neutrons striking the target in an experiment like that shown in Fig.1 ordinarily pass through the target without interacting with any of the nuclei. The number which does collide is found to be proportional to the beam intensity, to the atom density *N* of the target and to the area and thickness of the target. These observations can be summarized by the equation.

$$\text{Number of collisions per second} = \sigma I N a X \qquad (1.2)$$

where σ, the proportionality constant is called the *cross section*. The factor NaX in Eq. (1.2) is the total number of nuclei in the target. The number of collisions per second with a single nucleus is therefore just σI. It follows that *σ is equal to the number of collisions per second with one nucleus per unit intensity of the beam.*

There is another way to view the concept of cross section. As already noted, a total of *Ia* neutrons strike the target per second. Of these, σI interact with any given nucleus. It may be concluded therefore that

$$\frac{\sigma I}{aI} = \frac{\sigma}{a} \qquad (1.3)$$

is equal to the probability that a neutron in the beam will collide with this nucleus. It will be clear from Eq. (1.3) that σ has units of area. In fact, it is not difficult to see that σ is nothing more than the *effective cross-sectional are*a of the nucleus, hence the term "cross-section". Neutron cross sections are expressed in units of barns, where 1 barn, abbreviated b, is equal to $10^{-24}cm^{2}$. One thousandth of a barn is called a millibarn, denoted as mb.

Up to this point it has been assumed that the neutron beam strikes the entire target. However, in many experiments the beam is actually smaller in diameter than the target. In this case, the above formulas still hold, but now α refer to the area of the beam instead of the area of the target. The definition of cross section remains the same, of course.

Each of the processes described in section 1.1 by which neutrons interact with nuclei is denoted by a characteristic cross section. Thus elastic scattering is described by the *elastic scattering cross section* σ_s*;* inelastic scattering by the *inelastic scattering cross section,*σ_i*;* the (n, γ) reaction (radiative capture) by the capture cross section,σ_γ; fission by the fission cross section, σ_f; etc. The sum of the cross sections for all possible interactions is known as the total cross section and is denoted by the symbol σ_i; that is,

$$\sigma_i = \sigma_s + \sigma_{i+} \sigma_\gamma + \sigma_f \ \dots \dots \qquad (1.4)$$

The total cross section measures the probability that an interaction of any type will occur when neutrons strike a target. The sum of the cross sections of all absorption reactions is known as the *absorption cross section* and is denoted by σ_a. Thus

$$\sigma_a = \sigma_\gamma + \sigma_f + \sigma_p + \sigma_z + \cdots \qquad (1.5)$$

Where $\sigma_p and \sigma_\alpha$ are the cross sections for the (n, p) and (n, α) reactions. As indicated in equation (1.5), fission, by convection, is treated as an absorption process.

To return to equation (1.2), this can be written as

Number of collisions per second (in entirely target) = $IN\sigma_t \times aX$ (1.6)

where σ_t has been introduced because this cross section measures the probability that a collision of any type may occur, since αX is the total volume of the target, it follows from equation (1.6) that the number of collisions per cm^3/sec in the target, which is called the *collision density F* is given by

$$F = IN\sigma_t \quad (1.7)$$

The product of the atom density *N* and a cross section, as in equation (3.7), occurs frequently in the equations of nuclear engineering; it is given the special symbol □, and is called the *macroscopic cross section.* In particular, the product $N\sigma_t = \Sigma_t$ is called the *macroscopic total cross section*, $N\sigma_s = \Sigma_s$ is called the *macroscopic scattering cross section*, and so on. Since $N\ and\ \sigma$ have units of cm^{-3} and cm^2, respectively, □ has units of cm^{-1}. In terms of the macroscopic cross section, the collision density in equation (1.7) reduces to

$$F = I\Sigma_t \quad (1.8)$$

SELF ASSESSMENT TEST 2

(i) Define the following terms:

- Cross section
- Absorption cross section

- Macroscopic cross section

(ii) A beam of 1-MeV neutrons of intensity 5×10^8 neutrons/cm^2-sec strikes a thin ^{12}C target. The area of the target is $0.5cm^2$ and it is 0.05cm thick. The beam has a cross sectional area of $0.1cm^2$. At 1-MeV, the total cross section of ^{12}C is 2.6b.

(a) At what rate do interactions take place in the target?

(b) What is the probability that a neutron in the beam will have a collision in the target?

1.3 CONCLUSION

In conclusion we have been able to examine different ways that neutrons can interact with nuclei of atoms as well as neutron dynamics.

1.4 SUMMARY

In this unit, we have been able to understand that neutrons have the ability of interacting with nuclei of atoms and the different forms of interactions these neutrons can have with nuclei.

1.5 TUTOR MARKED ASSIGNMENTS

(i) The scattering cross sections (in barns) of hydrogen and oxygen at 1-MeV at 0.0253eV are given in the table below. What are the values of σ_s for the water molecule at these energies?

	1-MeV	0.0253eV
H	3	21
O	8	4

(ii) The value of σ_γ for ^1H at 0.0253eV is 0.332 b. what is σ_γ at 1eV?

(iii) A 1-MeV neutron is scattered through an angle of 45^0 in a collision with a ^2H nucleus.

(iv) What is the energy of the scattered neutron?

(v) What is the energy of the recoiling nucleus?

(vi) How much of a change in lethargy does the neutron undergo in this collision?

1.6 REFERENCES/FURTHER READING

W. Greiner and J.A. Maruhn, Nuclear Models by Springer.

UNIT 2 THERMALIZATION

CONTENTS

2.0 INTRODUCTION

Here we will discuss details of nuclear reactions in which neutrons are the projectiles. A good source of neutrons is α-particles bombarding light elements.

$$ {}^{4}_{2}He + {}^{9}_{4}Be \rightarrow {}^{12}C + {}^{1}_{o}n + Q $$

The α-particles are usually obtained from radium (Ra) in normal radioactive processes. In such a reaction, up to 10^7 neutrons are emitted with energy 1-13MeV. Neutrons with this kind of energy are referred to as *fast neutrons*, for the purpose of nuclear fission and large scale release of atomic energy; neutrons of energy in the neighbourhood of 0.0025eV are required. The neutrons are called *thermal neutrons*, because this is about the thermal energy ($^{3}/_{2}\,KT$) of molecules at thermal temperature. The process of slowing down fast neutrons is called ***"Thermalization or Moderation"***.

2.1 OBJECTIVES

After going through this unit, you will be able to:

- Explain the slowing down of neutrons in materials
- Explain the choice of various materials for slowing down processes.

2.2 MAIN CONTENTS

2.2.1 NEUTRON MODERATION

The process of getting fast neutrons to slow down to thermal neutron is called *moderation* or *thermalization*. This is achieved by passing the fast neutrons through some suitable material called *moderators*. In such a way that neutrons are not lost by absorption, but merely have their kinetic energy reduced progressively by elastic collision with the nuclei of the material, examples of good moderators are graphite and water. There are usually two frames of reference in the study of the dynamics of neutrons. They are as follows:

i. Laboratory frame

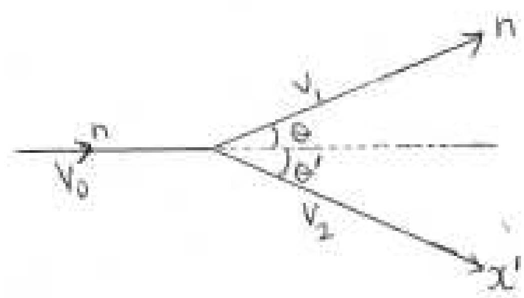


Fig.1 Laboratory frame of reference

In this frame, the target nucleus is at rest before the collision and it is approached by a projectile neutron with velocity V_o. After collision, the nucleus is scattered through angle θ and the target nucleus moves through angle θ'. This can be represented diagrammatically as shown above.

ii. The Centre of Mass Frame

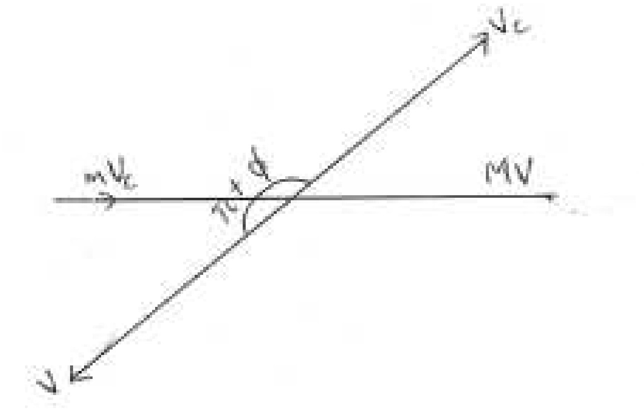
This is theoretical but a very useful approach in dealing with the dynamics of the moderation process. In this frame, the centre of mass of the neutron and the target nucleus is at rest and it is approached in opposite direction by the neutron and the target nucleus. According to the principle of conservation of momentum

$mV_c - MV = 0$

$$\text{or } V_c = \frac{MV}{m} \qquad (1)$$

Where V_c is the velocity of neutron; V is the velocity of target nucleus.

After collision, a neutron scatters off at an angle □ and the nucleus move all through $(\phi + \pi)$. In order to conserve momentum, velocity after collision remains unchanged.



Combining these two frames of references, we obtain a situation as in Fig.2 below:

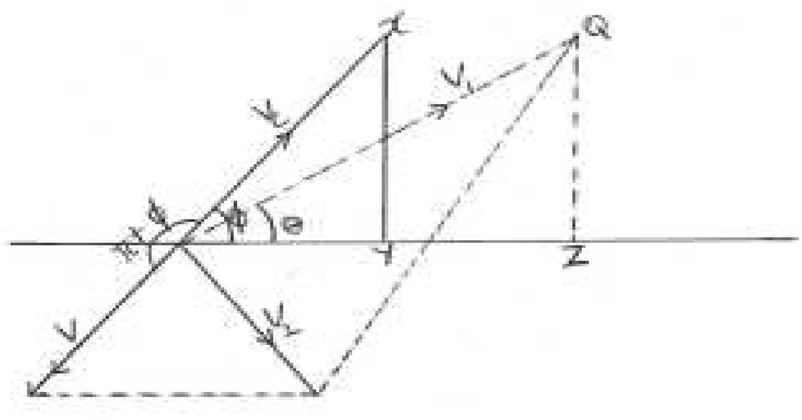


Fig.2 The centre of mass frame of reference

XY = QZ

$$\sin\theta = \frac{QZ}{V_1}$$

$V_1 \sin\theta = QZ$

$V_c \sin\phi = XY$

$$\frac{V_1}{V_c} = \frac{Sin\phi}{Sin\theta} \quad (2)$$

In the laboratory frame, the relative velocity = V_o since M is at rest. In the cantered mass frame, relative velocity = Vc + V since they approach opposite direction.

These two relative velocities must be equal in the 2 frames, in other words,

$V_o = V_c + V$………………

Combing equation (1) and (2) and eliminating V_c

$$V = \frac{mV_o}{M + m}$$

And $V_c = \dfrac{MV_o}{M + m}$ (eliminating V)

$$\frac{m}{M} \approx \frac{1}{A}$$

Where, A is the mass number of the nucleus

Therefore:.

$$V = \frac{V_0}{1 + A}$$

$$V_o = V(1 + A)$$

Such that $Vc = \dfrac{V_o A}{1 + A} and \dfrac{V_c}{V} = \dfrac{V_o A}{1 + A} \bullet \dfrac{1 + A}{V_o} = A$

From the diagram

$$V_1^2 = V^2 + V_c^2 - 2VVcCos(180^0 - \phi)$$
$$= V^2 + V_c^2 + 2VVcCos\phi$$
$$= V^2\left[1 + \frac{Vc^2}{V^2} + \frac{2VcCos\phi}{V}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$
$$V_1^2 = V^2(1 + A^2 + 2ACos\phi)$$

So,

Let the incident energy of the neutron from the laboratory frame be given as:

$$E_o = \frac{1}{2} mV_o^2 .$$

After the first collision, the energy is reduced to.

$$E_1 = \frac{1}{2} mV_1^2$$

And the fractional energy is given

$$\frac{E_1}{E_o} - \frac{\frac{1}{2}MV_1^2}{\frac{1}{2}mV_o^2} = \frac{V_1^2}{V_o^2}$$

$$\frac{E_1}{E_o} = \frac{\left[1 + A^2 + 2Cos\phi\right]}{\left[1 + A\right]^2} \quad \text{(Fractional energy)}$$

Cases of Interest

1. Glancing collision; (∅ = 0)

$$\frac{E_1}{E_o} = \frac{1 + A^2 + 2Cos\phi}{\left[1 + A\right]^2} = \frac{1 + A^2 + 2A}{1 + A^2 + 2A} = 1$$

This implies $E_1 \cong E_o$ therefore the neutron loses little or no energy on colliding with the nucleus.

2. Head on collision, (∅ = π)

Where, Cosπ = - 1

$$\frac{E_1}{E_o} = \frac{1 + A^2 - 2A}{(1 + A)^2} = \frac{(A - 1)^2}{(A + 1)^2}$$

This type of collision leads to maximum energy loss by the neutron e.g. for graphite which is a carbon allotrope and with its atomic mass, A = 12.

$$\frac{E_1}{E_o} = 72\%$$

3. General case: $0 < \emptyset < \pi$

Introducing $\alpha = \left[\frac{A-1}{A+1}\right]^2$

$(\Delta E)max = (E_o - E_1)max$

$= E_o(1 - E_1/E_o)$

$= E_o\,(1 - \alpha)$

$$\left(\frac{\Delta E}{E_o}\right)_{\max} = (1-\alpha)$$

$$= 1 - \left(\frac{A-1}{A+1}\right)^2$$

$$= \frac{4}{A} - \frac{8}{A^2} + \frac{12}{A^3} - \frac{16}{A^4} + \frac{20}{A^5}$$

$$\left(\frac{\Delta E}{E_o}\right)_{\max} = \frac{1-(1-\frac{1}{A})^2}{(1+\frac{1}{A})^2}$$

$$= \frac{4}{A}\left[1 - \frac{2}{A} + \frac{3}{A^2} - \frac{4}{A^3} + \frac{5}{A^4}\right]$$

A very important point is that for a material to serve as a good moderator the fractional energy loss must be large and from the expression above, the loss is smaller. From the above expression, it is observed that the fractional energy loss $\left[\frac{\Delta E}{E_o}\right]_{\max}$ is inversely proportional to the atomic mass of the moderator.

SELF ASSESSMENT TEST 1

(i) List and briefly explain the frames of references considered for neutron moderation.

(ii) Derive the fractional energy $\frac{E_1}{E_0} = \frac{[1 + A^2 \ +2A\cos\theta]}{[1 +A]^2}$

2.2.2 PASSAGE OF A BEAM OF NEUTRON THROUGH A MODERATING MATERIAL

In practice, we deal with a flux of up to 10^8 fast neutrons to be moderated by many nuclei of the moderator. We introduced the probability that a single neutron will be scattered through angle Ø lying between Ø and dØ such that the energy of the neutron after scattering lies between E + dE. The range of the energy is $E_1 = E_o$ for glancing angle collision and $E_1 = \alpha E_o$ for head on collision. The probability that a neutron will have energy E where αEo is less than E and less than E_o ($\alpha E_o<E<E_o$) after an arbitrary scattering is P(E). The energy between this range is:

$E-\alpha E_o = E_o\,(1-\alpha)$ so that

$$P(E)dE = \frac{dE}{E_o(1-\alpha)}$$

Normalizing this probability

$$\int_{E1=\alpha Eo}^{Eo} P(E)dE = \int_{\alpha Eo}^{Eo} \frac{dE}{\mathrm{E}_0(1-\alpha)} = 1$$

The average energy of a neutron after series of scattering is the probability that a single collision will have energy E. Then, the average energy is given by:

$$\langle E\rangle = \frac{\int_{\alpha E_0}^{E_0} EP(E)dE}{\int_{\alpha E_0}^{E_0} P(E)dE}$$

$$= \frac{1}{2E_0(1-\alpha)}\left[E^2\right]_{\alpha E_0}^{E_0}$$

$$\langle E\rangle = \frac{1}{2}Eo(1-\alpha)$$

Where $\alpha = \frac{(A-1)^2}{(A+1)^2}$

- ***Average log energy decrement (ξ)***

This term is introduced to obtain information about the average number of collisions which a fast neutron will make before its energy E_o is reduced to thermal energy E_t, when E_o is reduce to E, the log energy decrement is given by:

$$\log_e E_o - \log_e E \Rightarrow \log_e\left(\frac{E_o}{E}\right)$$

Average log decrement = $< \log_e\left(\frac{E_o}{E}\right) >$

$$\zeta = \int_{\alpha Eo}^{Eo} \log_e\left(\frac{E_o}{E}\right) P(E)dE$$
$$= \int_{\alpha Eo}^{Eo} \log_e\left(E_o / E\right)\frac{dE}{E_o(1-\alpha)}$$

Since $\int_{\alpha Eo}^{Eo} P(E)dE = 1$

Take $x = \frac{E}{E_o}$

Then for E = αE_o

Where $x = \alpha$

For E = E_o, $\Rightarrow x = 1$

dE = $E_o dx$ and $\log\frac{Eo}{E} = -\log x$

$$\zeta = -\frac{1}{1-\alpha}\int_{\alpha}^{1}\log x dx$$

$$= 1 + \frac{\alpha}{1-\alpha}\log\alpha$$

substitute

$$\alpha = \left(\frac{A-1}{A+1}\right)^2$$

$$\zeta = \frac{1 + \left(\frac{A-1}{A+1}\right)^2 \log\left(\frac{A-1}{A+1}\right)^2}{1 - \left(\frac{A-1}{A+1}\right)^2}$$

$$= 1 - \frac{(A-1)^2}{2A}\log_e\left(\frac{A-1}{A+1}\right)$$

For A > 1, a convenient approximation

$$\zeta = 2/A + \frac{2}{3} \Rightarrow$$

$$\zeta \propto \frac{1}{A}$$

$$n = \frac{1}{\zeta}\log_e\left(\frac{E_o}{E}\right)$$

Where n is the number of collision required to reduce fast neutron energy E_o to thermal energy E_t.

- ***Slowing down power Sp***

The effectiveness of a moderating material is not only by log energy decrement but also by the density of the substance, that is, the number of colliding centres that a material has per unit volume.

The material must also have small *absorption cross section,* σ_a. Slowing down power is a term which combines the various parameters and it is defined as:

$$S_p = \xi N \sigma_a$$

Or

$$S_p = \frac{\xi N_a \rho \sigma_a}{A}$$

Where ρ is the density

A is the atomic weight of the moderator

σ_a = Absorption cross section

N_a = Avogadro's constant

- ***Neutron interaction***

All neutrons at the time of their birth are fast. In penetrating through matter, they undergo characteristics process of energy degradation or moderation. The probability of a neutron interaction with a nucleus taking place in the moderating medium is the cross-section represented by σ. This is measured as effective area presented to the neutron and it is expressed in unit of barns.

Where 1 barn = 1 x $10^{-24}cm^2$ (total part of material presented for interaction).

The total cross-section σ_t has several components:

$\sigma_t = \sigma_{et} + \sigma_{inelastic} + \sigma_{ab} + \sigma_f + \ldots..$

which is the sum of elastic, inelastic, absorption and fusion cross-section, all of which are strongly energy dependent. The total cross-section µt is a microscopic quality. When we multiply by the number N of the absorber atoms per unit volume we have

$$\sum = \sigma N$$

The removal of neutrons from a beam which transverses a thickness t

$$I = Ioe^{-\sigma Nt}$$

Or

$I = Ioe^{\Sigma t}$

I_o – initial intensity of the beam neutron

THE MEAN FREE PATH is the distance travelled by the neutron before collision

$\pi = 1/\Sigma$ or $\frac{1}{\sigma N}$

- ***Absorption***

As neutrons approach thermal energy, the likelihood of capture by absorber nucleus increases i.e. the absorption cross-section increases σ_{ab}. For many absorbing nucleic, as neutron energy become very small (0.01 to 10, 000eV), the absorption cross-section is directly proportional to the inverse of the velocity and inversely proportional to the energy.

$$\sigma_{ab} \alpha \frac{1}{v} + \frac{1}{\sqrt{E}} \qquad (1)$$

Between the range of energy (0.001 to 10, 000eV) if a neutron enters a reactor or start in a reactor with certain energy E_o and having energy E after a certain number of the collision.

$$\frac{\sigma_o}{\sigma} = \frac{\sqrt{E}}{\sqrt{E_o}} \qquad (2)$$

From the absorption spectrum of neutrons inside a reactor, resonance effect is observed.

EXAMPLE

Cadmium has resonance effect of 0.176eV

When neutron is absorbed by a material, there are many channel of decay i.e. (n, γ), (n, p), n, n), (n, α). This implies that the first letter in the bracket is absorbed and the other is emitted.

$${}^{A}_{Z}X + {}^{1}_{o}n \rightarrow {}^{A-Y+1}_{z}Y + yn^{1}_{o}$$

Such that the first reaction (n, d) the absorption cross-section can be written as:

$$\sigma_{(n,\gamma)} = \frac{\lambda^2 \eta_n \eta_\gamma}{4\pi\left[(E - E\gamma)^2 + \frac{1}{4}\eta^2\right]}$$

Where η_n = partial width for (n, n) reaction

η_γ = partial width for (n, γ) reaction

$\eta = \eta_n + \eta_\gamma$ = width of resonance at ½ of its height

λ = de Broglie's wavelength for neutrons of energy E

E = energy at which absorption cross-section is calculated

SELF ASSESSMENT TEST 2

(i) Define the following terms as related to Neutron Physics:

- Slowing Down Power
- Average Log Energy Decrement
- Cross section in moderating process
- Resonance effect

(ii) In an experiment to measure the total cross-section of lead for 10MeV neutrons, it is found that 1cm thick lead attenuate neutron flux to 84.3% of its initial value. The atomic weight of lead is 207.21 and its specific gravity is 11.3.Calculate the total cross-section

(iii) Estimate the probability of (n, n) and (n, γ) in indium, known to have a neutron resonance at 1.44eV with an absorption η, of 0.1eV and σ, cross-section of 28, 000barns.

2.3 CONCLUSION

In conclusion we have been able to examine how fast neutrons are reduced to thermal neutrons such that they can be useful in the reactor to produce energy.

2.4 SUMMARY

In this unit, we have been able to understand that neutrons have the ability being thermalised. Also, we studied various parameters used in studying this process such as Slowing Down Power and Average Log Energy Decrement

2.5 TUTOR MARKED ASSIGNMENTS

(i) Explain a resonance effect that is observed in a reactor

(ii) Determine the thermal energy of a neutron E_t

(iii) Discuss hydrogen as a special moderator (why is it special using the formula we just derived)

(iv) If the number of neutrons is reduced to half of its original value after passing through a moderating material (heavy water) of thickness 15cm. Calculate its thermal diffusion Area and length. Given the Diffusion co-efficient and Absorption cross section of heavy water as 0.87cm and $2.9X10^{-5}cm^{-1}$ respectively.

2.6 REFERENCES/FURTHER READIN

R. Gautreau and W. Savin, Schaum's outline of theory and problems of Modern Physics, 1999 edition.

UNIT 3 TRANSPORT AND DIFFUSION EQUATION

CONTENTS

3.0 INTRODUCTION

It is essential to know the spatial and energy distributions of the neutrons in a field in a nuclear fission reactor, D–T (or D–D) fusion reactor, or other nuclear reactors populated with large numbers of neutrons. It is obvious why the spatial distribution should be known, and because neutron reactions vary widely with energy, the energy distribution is also a critical parameter. The neutron energy distribution is often called the *neutron spectrum*. The neutron distribution satisfies transport equation. It is usually difficult to solve this equation, and often approximated equation so called diffusion equation is solved instead. In this unit an overview of transport equation and diffusion equation of neutrons are presented.

3.1 OBJECTIVES

After going through this unit, you will be able to:

- Explain the behaviour of neutrons in the reactor

- Explain the physics of neutrons in a reactor in terms of the equation of continuity and diffusion equation.
- Deduce the diffusion length of neutrons and the buckling factor of the reactor.

3.2 MAIN CONTENT

3.2.1 FICK'S LAW

The original Fick's law states that the rate of change of a solute concentration is proportional to the negative gradient of the solute concentration (which was originally used to account for chemical diffusion).

Neutrons behave in much the same way as a solute in a solution. This means that if the density (flux) of neutron is higher in one part of a reactor than at another, there is a net flow of neutron into the region of lower neutron density. For example, suppose that the flux (it is usual in nuclear engineering to make calculations with the flux, which is proportional to neutron density, rather than with the density itself) varies along the *x-direction.* As shown in Fig. 3.1. Then Fick's law is written as

$$J_x = -D\frac{d\varphi}{dx} \quad (3.1)$$

In this expression J_x is equal to the net number of neutrons which pass per unit time through a unit area perpendicular to the *x-direction*; it has the same units as flux, namely, neutrons/cm^2-sec. The parameter D in Eq. (3.1) is called the *diffusion coefficient.*

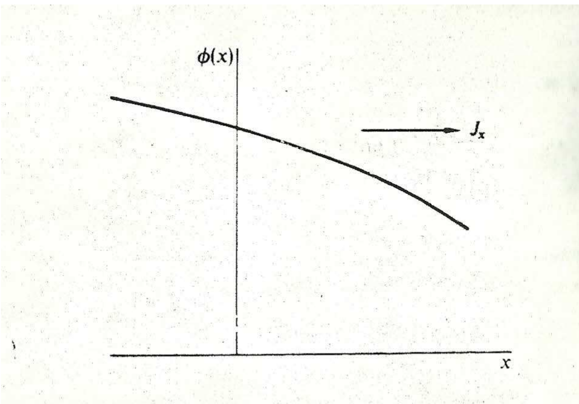


Fig. 3.1 Neutron flux and current

Equation (3.1) shows that if, as in Fig.3.1, there is a negative flux gradient, then there is a net flow of neutrons along the positive *x-direction* as indicated in figure.

To understand the origin of this flow, consider the neutrons passing through the plane at $x=0$. These neutrons pass through the plane from left to right as a result of collisions to the left of the plane; and, conversely, they flow from right to left as the result of collisions to the right of the plane. However, since the flux is larger for negative values of x, there are more collisions per cm^3/sec on the left than on the right. More neutrons are therefore scattered from left to right than the other way round, with the result that there is a net flow of neutrons in the positive *x-direction* through the plane, just as predicted by Eq. (3.1). It is important to recognize that the neutrons do not flow from regions of high flux to low flux because they are in any sense "pushed" that way. There are simply more neutrons scattered in one direction than in the other.

The flux is generally a function of three spatial variables, and in this case Fick's law is

$$J = -D \; grad \; \varphi = -D\nabla\varphi \qquad (3.2)$$

Here J is known as the *neutron current density vector*, and grad=∇ is the gradient operator. The physical significance of the vector $\boldsymbol{J}$ with a unit vector in the *x-direction* $\mathbf{a}_x$ this gives the x-component of $\mathbf{J}$, namely J_x :

$$\mathbf{J} . \boldsymbol{a}_x = J_x$$

which, as already noted, is equal to the net flow of neutrons per second per unit area normal to the *x-direction*. It follows, therefore, that if **n** is a unit vector pointing in an arbitrary direction then

$$\mathbf{J}.\mathbf{n} = J_n \tag{3.3}$$

Is equal to the net flow of neutrons per second per unit area normal to the direction of **n**.

Returning to Eqs.(3.1) and (3.2), it may be noted that, since J_x and **J** have the same unit as φ , *D* has units of length. It can be shown by arguments which are too lengthy to be reproduced here that *D* is given approximately by the following formula:

$$D = \frac{\lambda_{tr}}{3}, \tag{3.4}$$

where λ_{tr} is called the *transport men free path*, and is given in turn by

$$\lambda_{tr} = \frac{1}{\Sigma_{tr}} = \frac{1}{\Sigma_s(1-\bar{\mu})} \tag{3.5}$$

In this equation, Σ_{tr} is called the *macroscopic transport cross section,* Σ_s is the *macroscopic scattering cross section of the medium,* and $\bar{\mu}$ is the average value of the cosine of the angle at which neutrons are scattered in the medium. The value of $\bar{\mu}$ at most of the neutron energies of interest in reactor calculations can be computed from the simple formula

$$\bar{\mu} = \frac{2}{3A}, \tag{3.6}$$

where A is the atomic mass number of the medium.

It must be emphasized that Fick's law is not an exact relation. In particular, it is not valid

(i) In a medium which strongly absorbs neutrons.

(ii) within about three mean free paths of either a neutron source or the surface of a medium; and

(iii) when the scattering of neutrons is strongly anisotropic.

To some extent these limitations are present in every practical reactor problem. Nevertheless, as noted earlier, Fick's law and diffusion theory are often used to estimate reactor properties.

3.2.2. EQUATION OF CONTINUITY

The equation of continuity is the mathematical statement of the obvious fact that since neutron do not disappear unaccountably, a time rate of change in the number of neutron in a volume, V within a medium must be accounted for. In particular, it follows that

Rate of change of neutron, V = (Rate of production of neutron in the volume) - (Rate of absorption in the volume) – (Rate of leakage in the volume) ……………………………………………………………… (3.7)

If η is the density of neutrons at any point and time in volume v, then the total number of neutrons inside a volume, dv is then given as ηdv, but the total number of neutron inside the volume v is given as $\int\eta$dv. Therefore the rate of change in the number of neutron is given as $\int_v \frac{d\eta}{dt} dv$.

Also, let S be the rate at which neutrons are emitted from a source per cubic metre for volume, v. The rate at which neutrons are produced through v is given as:

$$\text{Production rate} = \int_v S dv$$

The rate at which neutrons are lost by absorption per cm^3/sec is given as $dv = \Sigma_a \varphi$ where $\sum_a$ is the absorption cross-section through the volume.

Therefore, the total number of neutron lost due to absorption is represented as

$$Absorption = \int_v \Sigma_a \varphi dv$$

Consider the flow of neutron in and out of volume ‘v’. If *J* is the neutron current density vector on the surface of v and n is a unit vector pointing outward from the surface. Then the dot product of J and n (J. n) is the net number of neutrons passing outward through the surface per cm^3 per second. It follows that the total rate of leakage of neutrons through the surface A is given as

$$\int_A J.n\, dA$$

Hence equation (3.7) becomes

$$V = \int_v \frac{\partial \eta}{\partial t} dv - \int_v \Sigma_a \phi dv - \int \nabla.J dv$$

$$\frac{\partial \eta}{\partial t} \equiv S - \Sigma_a \phi - \nabla.J \qquad (3.8)$$

If the neutron density is time dependent, then the expression density now becomes zero

$$0 \equiv S - \Sigma_a \phi - \nabla.J \qquad (3.9)$$

Equation (3.9) is called the steady state equation of continuity.

SELF ASSESSMENT TEST 1

(i) State Fick’s law.

(ii) Briefly explain the equation of continuity.

(iii) State the steady state equation of continuity and explain what each term stands for in the equation.

3.2.3. DIFFUSION EQUATION

The neutron diffusion equation is obtained by substituting the expression $J = -D\nabla\phi$ (where D is the diffusion coefficient) into equation (3.8) above. If D is not a function of space variable i.e *x,y,z*. This implies $D \neq D(x, y, z)$. Then we will have:

$$D\nabla^2\phi - \Sigma_a\phi + S = \frac{\partial\eta}{\partial t} \quad (3.10)$$

and $\phi = nv_i$ where v is the velocity of the neutron, when η is independent of time, the expression on the left hand side becomes zero i.e $\frac{\partial\eta}{\partial t} = 0$ and it is called the steady state diffusion equation. But if dependent on time, then it is called diffusion equation.

Dividing equation(3.10) through by D, then we will have

$$\nabla^2\phi - \frac{\Sigma_a\phi}{D} + \frac{S}{D} = 0 \quad (3.11)$$

Substituting

$\frac{\Sigma_a}{D} = \frac{1}{L^2}$ into equation (3.11)

$$\nabla^2\phi - \frac{1}{L^2}\phi + \frac{S}{D} = 0 \quad (3.12)$$

$$\therefore\ L^2 = \frac{D}{\Sigma_a} \quad (3.13)$$

The quantity *L* appears frequently in nuclear engineering problems and is called *diffusion length:* L^2 is called the *diffusion area.* Since *D* and Σ_a have units of cm and cm^{-1}, respectively, it follows from Eq.(3.13) that L^2 has units of cm^2 and L has units of cm. a physical interpretation of L and L^2 will be given later in this unit.

3.2.4. BOUNDARY CONDITIONS

To obtain neutron flux □ from the diffusion equation, it is necessary to specify certain *boundary condition* which must be satisfied by the solution i.e.

i. Φ must be real and non-negative function

ii. Φ must be finite except perhaps at singular points of a source distribution.

In many problems, neutrons diffuse in a medium which has an outer surface, that is, a surface between the medium and the atmosphere. In the immediate vicinity of such surface, Fick's law is not valid which means the diffusion equations is not valid there either. Exact (non-diffusion theory) calculations show, however, if the flux as calculated from the diffusion equation is assumed to vanished at a small distance d beyond the surface, then the flux determined from the diffusion equation is very nearly equal to the exact flux in the interior of the medium, though not, of course, near the surface. This state of affairs is illustrated in fig.3.2.

The parameter d is known as the *extrapolation distance*, and for most cases of interest it is given by the simple formula

$$d = 0.71\lambda_{tr} \qquad (3.14)$$

where λ_{tr} is the transport mean free path of the medium. From $D = \frac{\lambda_{tr}}{3}$, and so d becomes

$$d = 2.13D \qquad (3.15)$$

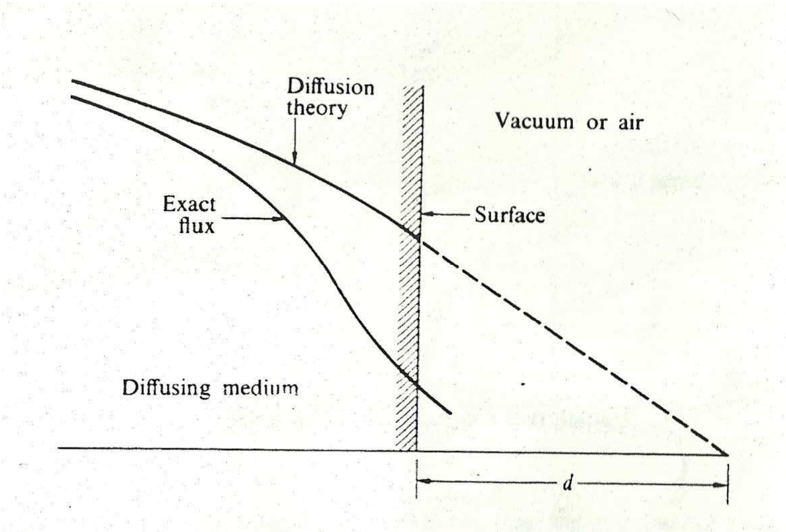


Fig.3.2 The extrapolation distance at a surface

Thus, from Eq. (3.15) it will be seen that *d* is usually small compared with most reactor dimensions. It is often possible, therefore, when solving the diffusion equation, to assume that the flux vanishes at the actual surface of the system.

Boundary conditions at an interface between two different media (e.g. between the reactor core and reflector) must also be specified. The conditions are that both the flux and the component of the current normal to the surface must be continuous across the boundary. Thus, at an interface between two regions A and B, we must have:

$$\varphi_A = \varphi_B \tag{3.16}$$

$$(J_A)_n = (J_B)_n \tag{3.17}$$

3.2.5 DIFFUSION LENGTH

It is of interest at this point to examine the physical interpretation of the diffusion length, which appears in the diffusion equation and in so many of its solutions. To this end, consider a monoenergetic point source emitting S neutrons per second in an infinite homogenous moderator. As these neutrons diffuse about in the medium, individual neutrons move in complicated, zigzag paths due to successive collisions as indicated in Fig. 3.3. Eventually, however, every neutron is absorbed in the medium- none can escape, since the medium is infinite.

The number of neutrons, *dn*, which are absorbed per second at a distance from the source between *r* and *r+dr* is given by

$$dn = \Sigma_a \phi(r) dV,$$

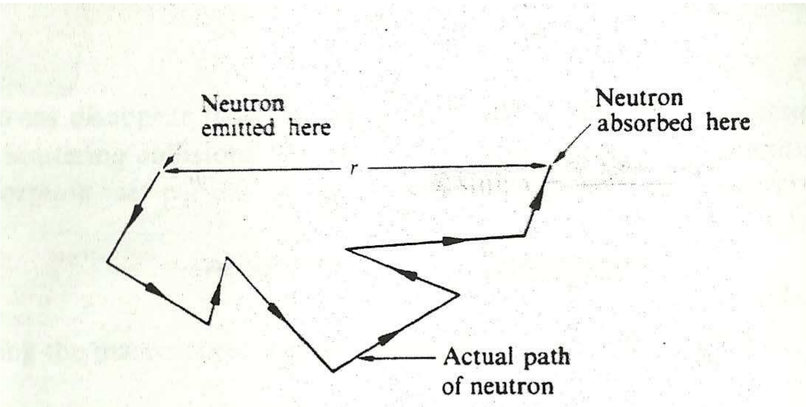


Fig. 3.3. Trajectory of a neutron in moderating medium

where $\phi(r)$ is the flux from the point source and $dV = 4\pi r^2\ dr$ is the volume of a spherical shell of radius *r* and thickness *dr*. Introducing $\phi(r)$ from $\phi(r) = \frac{Se^{-r/L}}{4\pi Dr}$ gives

$$dn = \frac{S\Sigma_a}{D} re^{-r/L}\,dr = \frac{S}{L^2} re^{-r/L}\,dr,$$

Since S neutrons per second are emitted by the source and *dn* are absorbed per second between *r* and *r+dr*, it follows that the probability *p(r) dr* that a source neutron is absorbed in *dr* is

$$P(r)dr = \frac{re^{-r/L}}{L^2} dr$$

It is now possible to compute the average distance from the source at which a neutron is absorbed, by averaging *r* over the probability distribution *p(r) dr*. For somewhat obscure reasons, however, it is more usual in nuclear engineering to compute the average of the square of this distance itself. Thus,

$$\overline{r^2} = \int_o^\infty r^2 \frac{(re^{-r/L})}{L^2}$$

$$\overline{r^2} = \frac{1}{L^2}\int_o^\infty r^3 e^{-r/L} dr$$

$$\overline{r^2} = 6L^2$$

$$\therefore L^2 = \frac{1}{6}\overline{r^2}$$

In words, the last equation states that L^2 is equal to one-sixth the average of the square of the vector ("crow-flight") distance that a neutron travels from the point where it is emitted to the point where it is finally absorbed. It follows from this result, that the greater the values of L, the further neutrons move, on the average, before they are absorbed. Measured values of *L* and L^2 for thermal neutrons will be discussed later.

SELF ASSESSMENT TEST 2

(i) State the diffusion equation and explain what each term of the equation stands for.

(ii) What are the conditions imposed on the diffusion equation to get the neutron flux?

(iii) Describe the Physics of the diffusion length with respect to the average of the square of the distance that a neutron travels.

3.3 CONCLUSION

In conclusion, we have been able to examine the different physical parameters that describe the behaviour of neutrons. Understanding this behaviours aid the design of nuclear reactors.

3.4 SUMMARY

In this unit, we have been able to understand that neutrons diffuse in the reactor which obeys the Fick's law for chemical reactions. Also, the equation of continuity and diffusion equation has been used to explain the diffusion of the neutrons. In order to solve these equations, boundary conditions were assigned to solve these problems which led to the derivation of the diffusion length.

3.5 TUTOR MARKED ASSIGNMENTS

(i) State three conditions under which the Fick's law is not valid.

(ii) Deduce the diffusion length and diffusion area from the diffusion equation.

(iii) The scattering cross section of carbon at 1eV is 4.8b. Estimate the diffusion coefficient of graphite at this energy.

(iv) It has been shown in this unit that the flux at the distance *r* from a point source emitting S neutrons per second in an infinite moderator is given by the formula below where L is a constant

$$\phi(r) = \frac{Se^{-r/L}}{4\pi Dr},$$

Find expression for

(a) the neutron current in the medium

(b) the net number of neutrons flowing out through a sphere of radius *r* surrounding the source.

3.6 REFERENCES/FURTHER READING

R. Gautreau and W. Savin, Schaum's outline of theory and problems of Modern Physics, 1999 edition.

UNIT 4 NUCLEAR REACTIONS

CONTENTS

4.0 INTRODUCTION

The main forces operating inside a nucleus are due to the volume term and surface term. This causes the nucleus to behave like a liquid drop. Therefore, if energy is applied from the outside, oscillation modes are excited in the same way as for a liquid drop. In this way, nucleuses can fission into two pieces, which is the process known as nuclear fission. Also, two or more light nuclei can fuse together to form heavy nuclide.

4.1 OBJECTIVES

After going through this unit, you will be able to:

- Explain how nuclear fission reaction produces neutrons from its chain reaction.
- Explain nuclear fusion as a thermonuclear reaction.
- Explain the criticality of a reactor

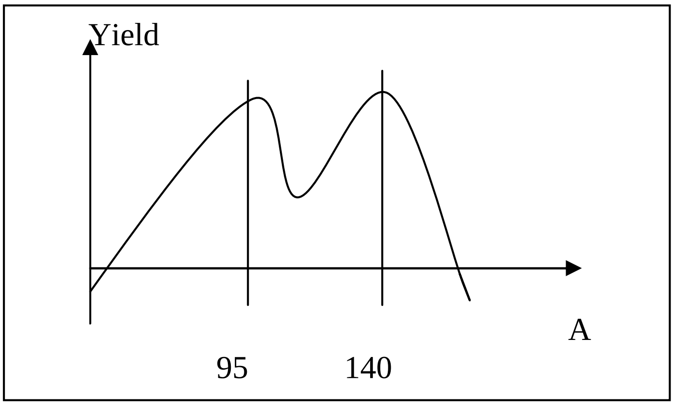
4.2 MAIN CONTENTS

4.2.1 NUCLEAR FISSION

When a nuclear fission takes place, two or more free neutrons are released. Fission is like any other nuclear reaction in which the total charge and the total number of nucleus must remain constant. e.g.

$$
\begin{aligned}
&{}^{235}_{92}U + {}^{1}_{0}\eta \rightarrow {}^{236}_{92}U \\
&\rightarrow {}^{A1}_{Z1}P_1 + {}^{A2}_{Z2}P_2 + K^{1}_{o}\eta
\end{aligned}
$$

Where K is the number of neutrons released. Usually $A_1 \neq A_2$. In other words, nuclear fission is usually asymmetric. Symmetric nuclear fission is one in which $A_1 = A_2$.



From the diagram above A_1 is about 95 and A_2 is about 140. From experiment result, the average value of K from this reaction is 2.5.

- ***Energy released from neutron yield***

We can calculate the energy released as follows:

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow \left({}^{236}_{92}U\right) \rightarrow {}^{98}_{40}Zr \xrightarrow{\beta^-} {}^{98}_{41}cf \xrightarrow{\beta^-} {}^{98}_{40}Mo + 2{}^{1}_{o}n$$

$$\rightarrow {}^{136}_{52}Te \xrightarrow{\beta^-} {}^{136}_{53}T \xrightarrow{\beta^-} {}^{136}_{54}Xe + 2{}^{1}_{o}n$$

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{98}_{42}Mo + {}^{136}_{54}Xe + 2{}^{1}_{o}n + 4{}^{0}_{-1}e$$

□m_i = 236.133 amu

□m_f = 235.905 amu

Δm = □mi - □mf

= 0.228 amu

= 0.228 amu x 931.5 MeV = 212.268 MeV

= 212.268x1.6 x 10^{-13}

= 3.36x10^{-11} J

In addition to this energy, some energy has been carried away by the emitted β^- particles and the γ rays. Fission process occurs sequentially releasing about 210MeV at each point of fission. Therefore if fission is allowed to continue, one can build a large amount of energy (3.36 x 10^{-11}J in a single fission) when this is multiplied by the Avogadro's number, we get the total energy released in 1 atomic mass i.e. in 235g of ${}^{235}U$

= 3.36 x 10^{-11} x 6.02x 10^{23}

= 2.02 x 10^{13}J

If this is consumed as fuel in one month in a nuclear power reactor, the power output P_{out}= $\dfrac{2.02\times10^{13}J}{30\times24\times3600}$

= 7.8 x 10^6 W

= 7.8 MeV

Lastly, this is more than the energy required by any University in the country.

4.2.2 THERMONUCLEAR REACTION (NUCLEAR FUSION)

Fusion is the synthesis of heavier nuclei from light ones and this can take place with the liberation of energy especially in cases where the total mass of the product is less than the total mass of the reactant i.e. $\Delta m > 0$ (implication).

□m_f - □m_i = $\Delta m > 0$

This is always the case for which $A_1 + A_2 < 60$ example:

$$^1_1H + ^1_1H \rightarrow ^2_1H + \beta^+ + 0.4MeV$$

The reactants have to be given high kinetic energy to overcome the coulomb's repulsive force between them, but the nuclear force of attraction takes over to fuse them together. In order to generate this kinetic energy, the temperature of the particles is raised and the energy is generated by thermal agitation e.g. to generate 480 KeV thermally, the temperature must be about 3.7x10^9 K. This is why the reaction is called thermonuclear reaction. Due to this high temperature requirement, no nuclear power plant is based on nuclear fusion.

Thermonuclear reaction can be accomplished by fusing proton and deuteron together $\left(^1_1H \ \& \ ^2_1H\right), \left(^1_1H \ \& \ ^1_1H\right), \left(^2_1H \ \& \ ^2_1H\right) etc$

The reaction involving the fusion of deuteron and tritium

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n + 17.6MeV$$

This reaction yields a large amount of energy and dissipates heat in the incredible short time of 1.2×10^{-6}s; thereby releasing a very high level of power. This is the basis of the “modern hydrogen bomb”. Fusion of light elements is known to be the source of “stellar energy”. In the interior of the sun, the temperature is high enough for nuclear fusion to occur.

SELF ASSESSMENT TEST 1

(i) Distinguish between nuclear fission and nuclear fusion.

(ii) Describe the conditions necessary for nuclear fusion to take place.

(iii) Mention one example each of physical phenomenon where nuclear fission and fusion can take place.

4.2.3 CRITICALITY OF A REACTOR

From the diffusion equation

$$D\nabla^2\phi - \Sigma_a\phi + S = 0$$

$$\Sigma_a = \square_{af} + \Sigma_{ac} \qquad (1)$$

The number of neutrons that will be absorbed by the fuel would be given as

$\Sigma_a = \Sigma_{af} + \Sigma_{ac}$ and not all neutrons absorbed by the fuel lead to fission.

If η is the probability of producing more neutron on average than the number of neutron emitted per neutron absorbed the total no of neutron emitted by the fuel is given as

$$= \eta\Sigma_{af}\phi \qquad (2)$$

Equation (2) can further be written as

$$\eta\Sigma_a\left(\frac{\Sigma_{af}}{\Sigma_a}\right)\phi$$

$$= \eta f\Sigma_a\phi \qquad (3)$$

This implies that

$$f = \frac{\Sigma_{af}}{\Sigma_a} = Utilization \;\; factor$$

Σ_{af} is absorption cross section by the fuel

Σ_a is total absorption cross section.

The ratio between the number of neutrons emitted to the number of neutrons absorbed is given as:

$$= \frac{\eta f\Sigma_a\phi}{\Sigma_a\phi} = K_\infty \qquad (4)$$

Since the number of neutrons absorbed = $\eta\Sigma_a\phi$: Therefore the total number of neutrons emitted and absorbed by fuel in the second medium is given as:

$$\eta f\Sigma_a\phi \text{ x } \Sigma_{af} = \eta f\Sigma_{af}\Sigma_a\phi$$

For the total of number of neutrons emitted by the fuel

$$\eta\times\left(\eta f\Sigma_{af}\Sigma_a\phi\right) = \eta^2 f\Sigma_{af}\Sigma_a\phi$$

From equation (4), equation (3) can be written as:

$$\eta f\Sigma_a\phi = K_\infty\Sigma_a\phi$$

Therefore, the diffusion equation can be written as:

$$D\nabla^2\phi - \Sigma a\phi + K_\infty\Sigma a\phi = 0$$

dividing through by D

$$\nabla^2\phi - \frac{1}{L^2}\phi + \frac{K_\infty \phi}{L^2} = 0$$

$$\nabla^2\phi + \frac{(K_\infty - 1)}{L^2}\phi = 0$$

$$Put\ \frac{K_\infty - 1}{L^2} as\ B^2$$

$$\nabla^2\phi + B^2\phi = 0$$

Where $B^2 = \frac{K_\infty - 1}{L^2}$

B is called "Buckling factor"

In a system where $K_\infty > 1$, the system is said to be "SUPER CRITICAL"

$K_\infty = 1$ the system is said to be "CRITICAL"

$K_\infty < 1$ the system is said to be "SUB CRITICAL"

The value of K_∞ can be controlled in a reactor e.g. to increase power in a reactor, K_∞ will be increased. The value of K_∞ cannot be controlled under explosive and nuclear reaction. Control rods e.g. carbon rods are used to control K in a system or reactor.

If K_∞ is great than 1, it means the number of fission increases from generation. In this case the energy released by the chain reaction increases with time. The system is said to be "Super critical: if $K_\infty < 1$, the number of fission decreases with time and the chain reaction said to be Sub Critical.
If $K_\infty = 1$, the chain reaction proceed at a constant rate, energy is released at a steady level, the system is said to be critical.

To increase the power being produced by a reactor, the operator increases K to a value greater than unity so that the reactor becomes super critical. When the desired power level has been reached, it returns the reactor to the critical by adjusting the valued of K to be unity and the reactor then maintains the specified power level.

To reduce power or shut the reactor down, the operator merely reduces K_1 making the reactor sub-critical and as a result, the output power at the system decreases.

SELF ASSESSMENT TEST 2

(i) Define the utilization factor in a nuclear reactor.

(ii) What do you understand by the criticality of a reactor?

(iii) Derive the buckling factor from the diffusion equation.

(iv) Briefly explain how K_∞ can be controlled in a nuclear reactor.

4.3 CONCLUSION

In conclusion, we have been able to examine the different forms of nuclear reactions. Also, we examined the dynamics of the neutron in the reactor.

4.4 SUMMARY

In this unit, we have been able to understand that nuclear reactions are of two forms (nuclear fission and fusion). We studied what each of these reactions entail as well as their practical applications. Also, we studied the neutron dynamics which led to defining the utilization factor and buckling factor. These factors assist to control reactions in a nuclear reactor.

4.5 TUTOR MARKED ASSIGNMENTS

(i) Briefly explain the process involved in a nuclear fission reaction.

(ii) What do you understand by the term neutron yield?

(iii) Explain the process involved in making an hydrogen bomb.

(iv) Briefly explain the different critical state of a nuclear reactor.

4.6 REFERENCES/FURTHER READING

R. Gautreau and W. Savin, Schaum's outline of theory and problems of Modern Physics, 1999 edition.

UNIT 5 NUCLEAR REACTOR

CONTENTS

5.0 INTRODUCTION

Devices which are designed so that fission reaction can proceed in a controlled manner are called "nuclear reactors". There are 2 main types of nuclear reactor.

a. Thermal reactor

b. Fast(Breeder) reactor

5.1 OBJECTIVES

After studying this unit, you will be able:

- Explain the different kinds of nuclear reactor
- Identify the components of a nuclear reactor
- Explain the function of each of these components

5.2 MAIN CONTENTS

5.2.1 CLASSIFICATION OF NUCLEAR REACTORS

5.2.1.1 THERMAL REACTORS

As mentioned earlier, for various neutron energies, reactions of different kinds and different sizes take place. If the energy of a neutron is lowered, its wavelike behaviour and the reaction cross-section will increase. The amount of this increase depends upon the nuclide and reaction. The reaction cross-section of a thermal neutron depends significantly upon the target nuclide. The cross-section of ^{235}U nuclear fission is shown in Figure 5.1. The nuclear fissions of $^{223}U, ^{239}Pu, and\ ^{241}Pu$ by thermal neutrons also have very large cross-sections. If the energy of a neutron is lowered to a thermal level, the achievement of criticality becomes easy. This is because as the neutron energy is decreased the cross-section for fission increases faster than the absorption cross-section of material in the reactor other than the fuel. A nuclear reactor with a large fission rate by thermal neutrons is called a thermal reactor.

As shown in Fig. 5.2, neutrons generated in nuclear fission have very high energy, and thus it is necessary to lower the energy in a thermal reactor. In order to moderate neutrons, the scattering explained in previous units can be used. In the high energy region, inelastic scattering and the (n, 2n) reaction can be used effectively. When the energy is lowered, however, these cross-sections are lost. Therefore, elastic scattering is the only useful moderation method over the wide energy region required to produce thermal neutrons. The rate of energy loss by elastic scattering is expressed by ξ. Light nuclides have higher values, as shown in Table 5.1. Thus, neutrons generated in fission collide with light nuclei and are moderated to thermal neutrons. Material used to moderate neutrons is called a moderator. A moderator should be a light nucleus, but at the same time, it should have a small neutron capture cross-section.

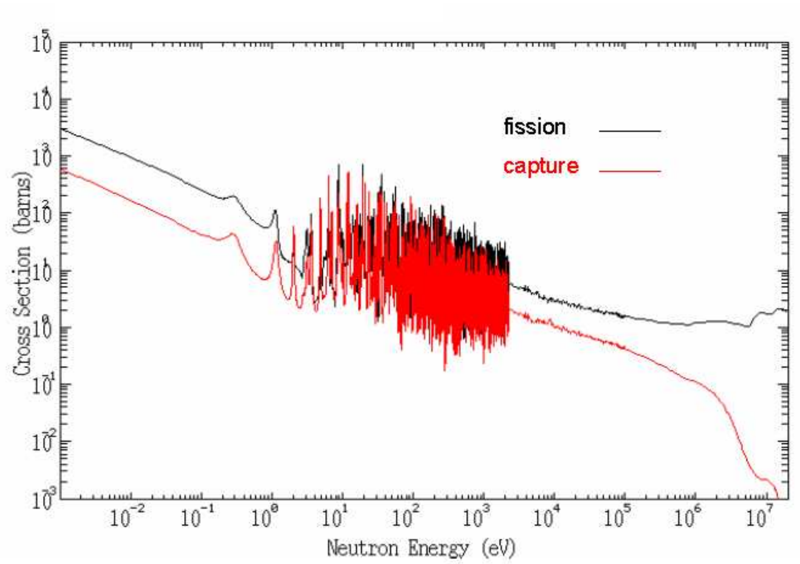


Figure 5.1 Fission and capture cross sections for ^{235}U

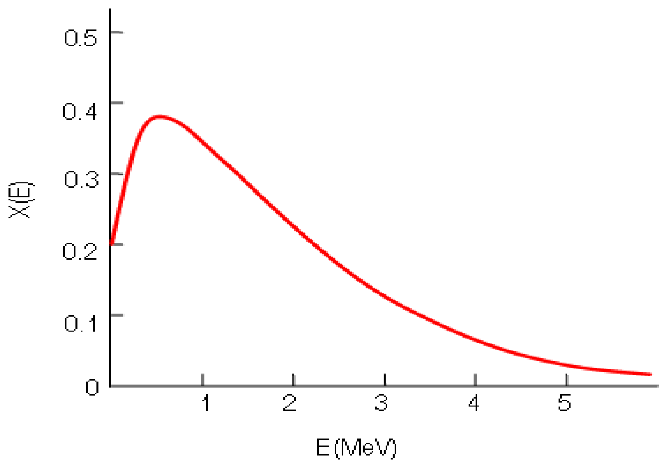


Fig. 5.2 Fission neutron energy spectrum

Moderator	A	α	ξ	Density (g/cm^3)	# of collisions From 2MeV To 1eV	$\xi\Sigma_s$ [cm^{-1}]	$\xi\Sigma_s/\Sigma_a$
H	1	0	1	gas	14	—	—
D	2	.111	.725	gas	20	—	—
H_2O	—	—	.920	1.0	16	1.35	71
D_2O	—	—	.509	1.1	29	0.176	5670
He	4	.360	.425	gas	43	1.6×10^{-5}	83
Be	9	.640	.209	1.85	69	0.158	143
C	12	.716	.158	1.60	91	0.060	192
^{238}U	238	.983	.008	19.1	1730	0.003	.0092

Table 5.1. Characteristics of typical moderator

As explained earlier, natural uranium contains only a small amount of fissile ^{235}U, with the rest being mostly ^{238}U. As shown in Fig. 5.3, ^{238}U shows a very large absorption for neutrons at energy of 6.67 eV. This type of absorption is called resonance absorption. We can see that there are also many resonance absorptions for energies higher than this. Therefore, if natural uranium is used as a fuel, many neutrons are absorbed by ^{238}U and it is difficult to make the nuclear reactor critical. The most direct method to solve this problem is to enrich ^{235}U. However, enrichment is very costly. Another good method is to allow suitable separation of fuel and moderator in the reactor.

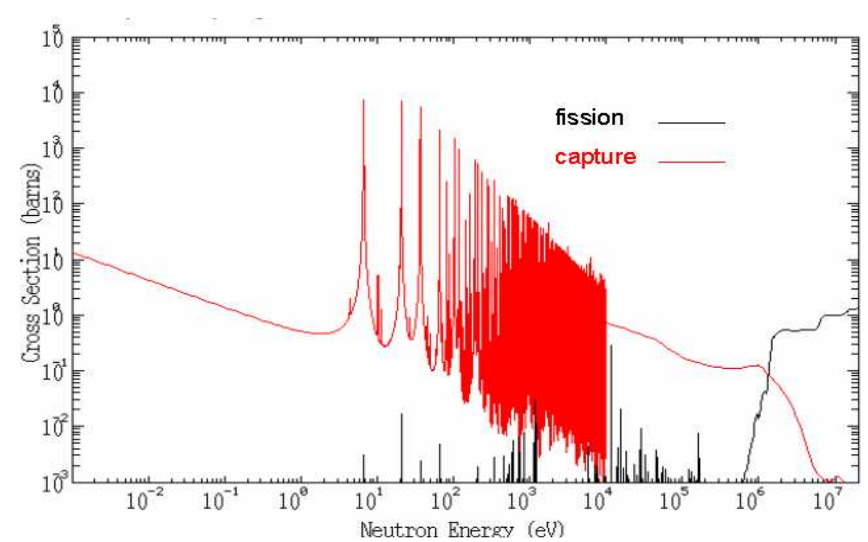


Fig.5.3. Cross sections for ^{238}U.

The separation of fuel and moderator is called a heterogeneous arrangement. Inside the moderator, both diffusion and moderation of neutrons take place. Since the nucleus of the fuel is heavy, it barely moderates neutrons, and thus, inside the fuel, diffusion is the dominant neutron motion. If the reactor is heterogeneous, neutrons scattered in the fuel tend to have successive collisions inside the fuel, and neutrons scattered in the moderator tend to have successive collisions inside the moderator.

Thus, neutrons generated by nuclear fission in the fuel are not moderated inside the fuel; instead they enter the moderator region and gradually lose their energy there by moderation. When a certain low energy is reached, most neutrons diffuse from the moderator region. In the energy region with large resonance absorption, neutrons diffusing from the moderator region to the fuel region are mostly absorbed on the surface of the fuel region, and thus they cannot enter the interior of the fuel region. This effect is called self-shielding. Most neutrons are finally moderated to thermal neutrons in the moderator region and enter the fuel region after repeated diffusion. They are then absorbed by ^{235}U. If the ratio between the fuel surface area and the volume is reduced by making broad fuel rods, self-shielding can greatly reduce resonance absorption. In light-water reactors, which are currently the most commonly operated reactors, neutron absorption by protons is considerable, enriched uranium is used and a heterogeneous arrangement is adopted.

5.2.1.2 BREEDER REACTORS

Thermal reactors have the following problems. When the neutron energy becomes large, the fission cross-section increases and the capture cross-section also increase at almost the same rate. The number of neutrons emitted when one neutron is absorbed in the nucleus expressed as η is:

$$\eta = v = \frac{\sigma_f}{\sigma_f + \sigma_c} \tag{5.1}$$

Here, σf and σc are the cross-sections for fission and capture, respectively, and v is the average number of emitted neutrons per nuclear fission. The value η depends on the energy of the colliding neutrons, as shown in Fig. 5.4. For thermal neutrons, η is about 2, however, when the energy is more than 0.1 MeV, η increases rapidly. If many neutrons are generated in this way, not only can a chain reaction be maintained, but it may also be possible that there will be excess neutrons.

If we let ^{238}U absorb these neutrons, ^{239}Pu can be produced, as mentioned before. That is, we can produce fissile material at the same time as we consume fissile material. The number of newly generated fissile atoms per consumed fissile atom is called the conversion ratio. If the value η is sufficiently larger than 2, it is possible to obtain a conversion ratio larger than 1. Thus, it is possible to generate more fissile material than is consumed. This is called breeding, and the conversion ratio in this case is often called the breeding ratio and this type of reactor is called a breeder reactor. In other words fissile material can be bred using fast neutrons in a fast breeder reactor.

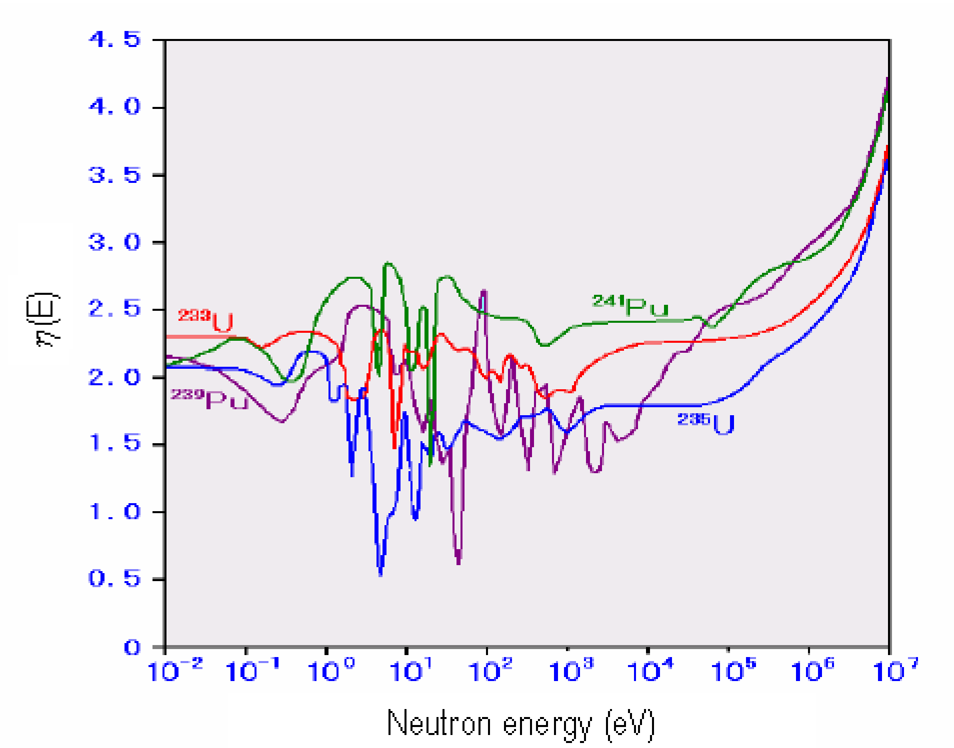


***Fig.5.4 η-values for important fissile nuclides*.**

Looking carefully at Figure 5.4, we can see that η for ^{233}U is larger than 2 in the region of thermal neutrons. Thus, it seems possible to perform breeding with thermal neutrons also. The nuclide ^{233}U can be prepared from ^{232}Th. However, since the margin of the neutron excess is very small in this case, ingenuity is required. A breeder reactor using thermal neutrons is called a thermal breeder reactor.

5.2.2 COMPONENTS OF NUCLEAR REACTORS

(1) **CORE:**

The central region of a reactor is called “CORE”. In a thermal reactor, this region contains the fuel, moderator and the coolant. The fuel includes the fissile isotope which is responsible both for the criticality of the reactor and for the release of fission energy. The fuel is some cases may also contain large and of fertilize material. Fertile materials are material which are themselves not fissile but from which fissile isotopes can be produced by absorption of neutrons example are ^{232}Th (from ^{233}U) which ^{233}U can be produced and ^{235}U from which ^{239}PU can be produced.

The moderator which is present only in thermal reactors is used to slow down and the neutrons from fission to thermal energy. Nuclei with low mass number are more effective for this purpose. Examples are water, heavy water and graphite.

The coolant is used to remove heat from the core and from other parts of the reactor where heat may be produced. Examples are, water, heavy metals, and various gases. With fast reactors, water and heavy water cannot be used as coolant since they can also slow down neutrons. Most fast reactors are cooled by liquid metal example liquid sodium.

(2) **BLANKET:**

This is a region of fertile material that surrounds the core. This region is designed specifically for conversion or breeding. Neutrons that escape from the core are intercepted in the blanket to enter the various conversion reactions.

(3) **REFLECTOR:**

This reduces the number of neutrons that finally leaves the reactor core. All of the neutrons do not return, but some do so as to save neutrons for the chain reaction in the reactor. Therefore a reactor with a reflector is dearly better than one with no reflector.

(4) **CONTROL RODS:**

These are movable pieces of neutron absorption material. They are used to control the reactor. Since they absorbed neutrons, any movement of the rods into or out of the reactor affects the multiplication factor K of the system. Withdrawal of the rods increases K, insertion decreases K. Thus the reactor can be started up, shut down, or its power output can be changed by the appropriate motion of the rods; for example Boron rods.

(5) **REACTOR VESSEL:**

All the components just described are located in the reactor vessel.

Water reactors, high-temperature gas-cooled reactors, and fast reactors are presently used for power generation or propulsion or are close to being in actual use. An overview of these reactors is given in Table 5.2.

	Moderator	Coolant	Neutrons	Breeding	Purpose	Status
Water reactor	water	water	thermal	no	power	practical use
HTGR*	graphite	He	thermal	no	multi-purpose	development
Fast reactor	none	Na	fast	yes	power	development

*High Temperature Gas Cooled Reactor

Table 5.2 Typical Nuclear Reactors

Water reactors can be either light-water reactors or heavy-water reactors. The light-water reactor is the predominant power reactor. Light-water reactors include boiling-water reactors (BWR, Figure 5.5), in which the coolant is boiled in the core, and pressurized-water reactors (PWR, Figure 5.6), in which boiling is suppressed under high pressure.

In a boiling-water reactor, power is generated by directly sending steam to a turbine. In a pressurized-water reactor, secondary cooling water is evaporated in a steam generator and the generated steam is sent to the turbine. Since the core in a heavy-water reactor is large, the moderator and coolant are generally segregated. The moderator is placed in an atmospheric vessel and the coolant is placed in a pressure tube. Either light water or heavy water is used as coolant. Usually, light water is used in the boiling-water type and heavy water is used in the pressurized-water type.

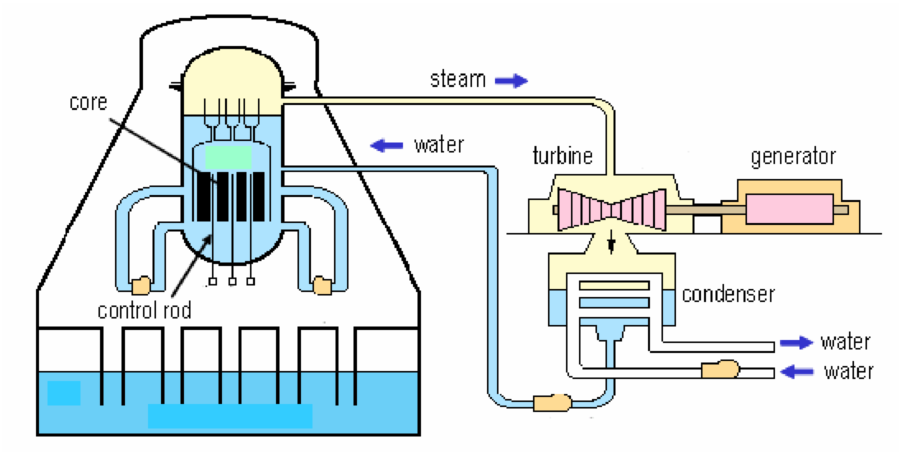


Fig.5.5 Boiling-water reactor (BWR).

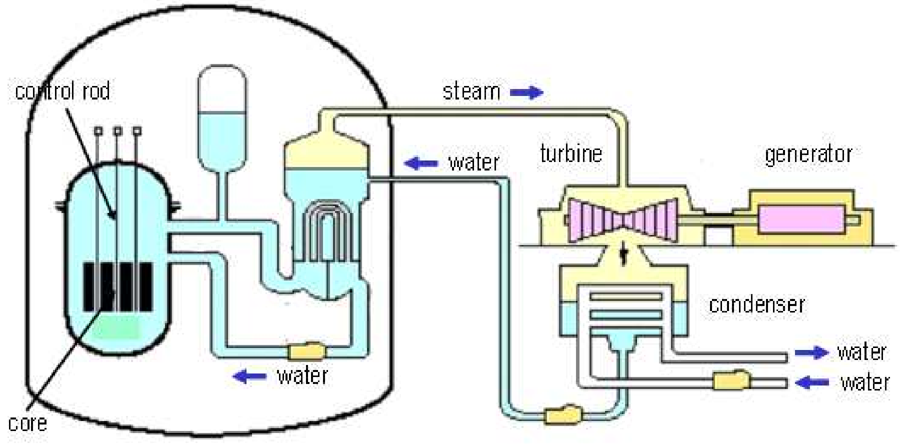


Fig.5.6 Pressurized-water reactor (PWR).

Currently under aggressive development is the high-temperature gas-cooled reactor, in which graphite is used as a moderator. Helium is used as coolant and higher temperatures can be obtained than are possible in other reactors. The high-temperature gas-cooled reactor is categorized into two types by the use of different fuel: the block-fuel type and pebble-bed type. Fig. 5.7 shows the pebble-bed reactor, developed as small modular reactors. Other reactors in which graphite is used as the moderator are the Calder Hall reactor, in which carbon dioxide is used as coolant, and the RBMK reactor, in which water is used as coolant. However, these reactors are gradually disappearing.

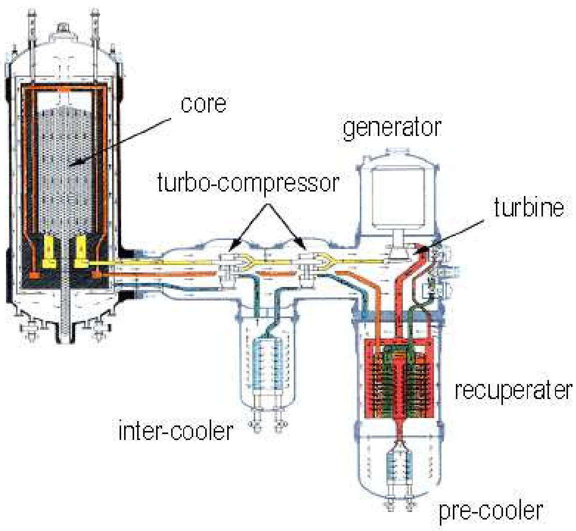


Fig.5.7 High-temperature gas-cooled reactor.

In most fast reactors (Fig. 5.8), a liquid metal, sodium, is used as coolant so that neutrons are not moderated. In these reactors, steam is also eventually generated. Since the contact of radioactive sodium and water is dangerous, a secondary sodium loop is installed between the primary sodium loop and the water. To avoid sodium-water reaction, lead or lead-bismuth is being experimented with as coolant. Another design uses gas such as helium or carbon dioxide as coolant since the density of gas is small and hardly reacts with neutrons.

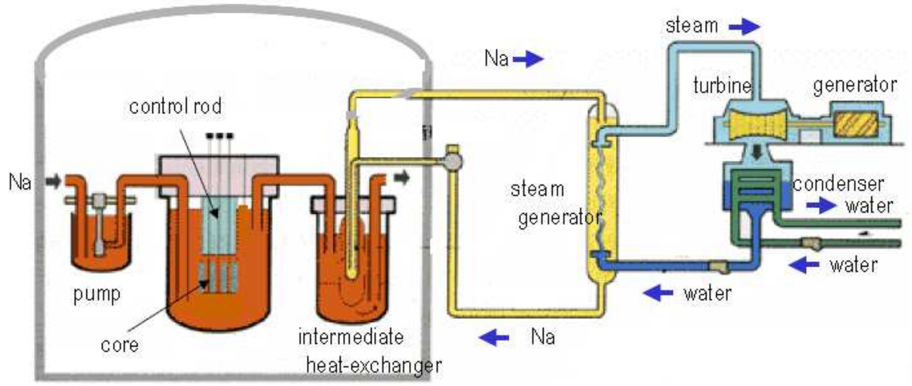


Fig.5.8 Fast breeder reactor (loop-type).

SELF ASSESSMENT TEST 1

(i) Name the classes of nuclear reactors

(ii) Explain briefly what happens to the reaction cross-section of neutron when their energies are lowered.

(iii) Explain briefly how to moderate generated neutrons from nuclear fission reaction in a reactor.

(iv) Give two properties of a moderator.

(v) List the problems of thermal reactors.

5.3 CONCLUSION

In conclusion, we have been able to examine the different classes of reactors. Also, we discussed about the components of a reactor as well as the nomenclature of reactors base on the moderator or coolant used in them.

5.4 SUMMARY

In this unit, we have been able to understand that there are two different classifications of reactors based on the kind of neutron used in them. Also we listed the components of a reactor and the function of each of these components in the reactor. We examined the present reactors used in the generation of power which were classified base on the coolant or moderator used in them.

5.5 TUTOR MARKED ASSIGNMENTS

(i) Explain the following terms:
- resonance absorption
- heterogeneous arrangement
- breeding
- conversion ratio

(ii) What is the difference between Fertile and Fissile material?

(iii) What are the components of a nuclear reactor?

(iv) List the components of a nuclear reactor.

(v) Give examples of moderators used in a nuclear reactor. Name the best of them all with reasons.

(vi) Give examples of reactors that are presently used for power generation.

5.6 REFERENCES/FURTHER READING

R. Gautreau and W. Savin, Schaum's outline of theory and problems of Modern Physics, 1999 edition.

SOLUTIONS AND ANSWERS

UNIT 1

1. Please see text

2 (i) Please see text

(ii) (a)according to table 11.3 in appendix II, N = 0.080×10^{24}. then from equation 1.2, the total interaction rate is $\sigma_t INaX$= $2.6 \times 10^{-24} \times 5 \times 10^{8} \times 0.080 \times 10^{24} \times 0.1 \times 0.05 = 5.2 \times 10^{5}$ interactions/sec. (ans.)

There are only two absorption reactions, namely, radiative capture and fission, which can occur when 0.0253-eV neutrons* interact with ^{235}U. The cross sections for these reactions are 99 b and 582 b, respectively. When a 0.0253-eV neutron is absorbed by ^{235}U, what is the relative probability that fission will occur?

(b) Since σ_γ *and* σ_f are proportional to the probabilities of radiative capture and fission, it follows that the probability of fission is $\sigma_f/(\sigma_\gamma + \sigma_f) = \sigma_f/\sigma_a = 582/681 = 85.5 percent.$

UNIT 2

1 (i) Please see text

(ii) Please see text

2 (i) Please see text

(ii) N – total number of atoms per unit volume of the moderating material interacting in the moderator. To calculate N = $\frac{Na}{A}$ x specific gravity (s.g)

N = Atomic density

$$N = \frac{6.02x10^{23}}{207.21} \times 11.3$$

$$\frac{I}{Io} = e - \sigma /(3.29 \times 10^{22} \times 1)$$

$$0.845 = e^{-\sigma(3.29 \times 10^{22} \times 1)}$$

$$\sigma = 3.1 \times 10^{-24} cm^2$$

$$\sum \; = \sigma N = 3.29 \times 10^{22} atoms / cm^3$$

(iii) σ = 28, 000barns

1barns = 10^{-24}cm^2

28000barns = 2.8x10^{-20}cm^2

Resonance occur when E = Eγ and η = ηγ

Recall: $$\sigma^{(n,\gamma)} = \frac{\lambda^2 \eta_n \eta_r}{4\pi\left[\left(E_0 - E_\gamma\right) + \frac{1}{4}\eta^2\right]}$$

$$\sigma^{(n,\gamma)} = \frac{\lambda^2 \eta_n \eta_\gamma}{\pi \eta^2}$$

$$recall\ \lambda = \frac{h}{p} = \frac{p}{\sqrt{2mE}}$$

Making $\eta_n \eta_\gamma$ the subject of the formula

$$\eta_n \eta_\gamma = \frac{\sigma_{(n,\gamma)} \pi \eta^2}{\lambda^2}$$

$$\eta_n \eta_\gamma = \frac{\sigma_{(n,\gamma)} \pi \eta_\gamma^{\ 2}}{\lambda^2}$$

$$\eta_n = \frac{\sigma_{(n,\alpha)} \pi \eta_\gamma}{\lambda^2}$$

$$\frac{\eta_n}{\eta_o} = \frac{\sigma_{(n,\alpha)} \pi}{\lambda^2} = 0.0153$$

UNIT 3

1 (i) Please see text

(ii) Please see text

(iii) Please see text

2 (i) Please see text

(ii) Please see text

(iii) Please see text

UNIT 4

1 (i) Please see text

(ii) Please see text

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2 (i) Please see text

(ii) Please see text

(iii) Please see text

(iv) Please see text

UNIT 5

1 (i) Please see text

(ii) Please see text

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